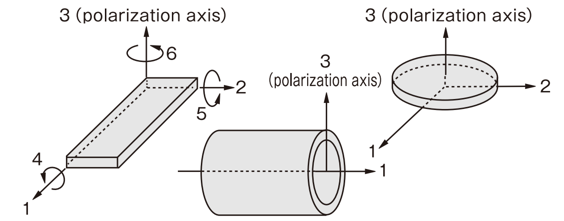
Piezoelectric properties of the material characteristics table

The piezoelectric property has shapes and the conditions of a direction. This condition is expressed in the vector and a symbol of tensor quantity. The superscript and subscript of this symbol have a meaning. Those outlines are summarized.



s_{11}
- Stress or strain is a direction of 1 axis.
- Strain or stress is a direction of 1 axis.

g_{33}
- Stress or strain is a direction of 3 axis.
- Normal direction of the electrode face is 3 axis.

c_{55}
- Shear stress is the rotatory direction of 2 axes.
- Shear strain is the rotatory direction of 2 axes.

d_{15}
- Shear strain is the rotatory direction of 2 axes.
- Normal direction of the electrode face is 1 axis.

d_{31}
- Strain or stress is a direction of 1 axis.
- Normal direction of the electrode face is 3 axis.

ε_{33}
- Dielectric displacement is a direction of 3 axis.
- Normal direction of the electrode face is 3 axis.

k_p, N_p : Constants of the radial direction of the disc which has orthogonal electrode face in a polarization axis.

k_t, N_t : Constants of the thickness direction of the plate which has orthogonal electrode face in a polarization axis.

k_{31}, N_{31}, d_{31}, g_{31} : Constants of the longitudinal direction of the rectangular plate which has orthogonal electrode face in a polarization axis.

k_{33}, N_{33}, d_{33}, g_{33} : Constants of the polarization direction of the virgulate which has orthogonal electrode face in a polarization axis.

k_{15}, N_{15}, d_{15}, g_{15} : Constants of the polarization axis (or orthogonal)-shear direction of the plate which has electrode face in a polarization axis.

$\varepsilon_{11}{}^T$: Free dielectric displacement of the plate which has electrode face in a polarization axis (at constant stress).

$\varepsilon_{33}{}^T$: Free dielectric displacement of the plate which has orthogonal electrode face in a polarization axis (at constant stress).

ε_0 : Electric constant. (=8.854×10^{-12} F/m)

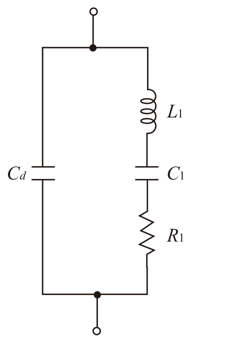
$Y_{11}{}^E$, $s_{11}{}^E$: Transverse elasticity of the electrode face direction of the plate which has orthogonal electrode face in a polarization axis (at constant electric field).

$Y_{33}{}^E$, $s_{33}{}^E$: Longitudinal elasticity of the polarization axis direction of the virgulate which has orthogonal electrode face in a polarization axis (at constant electric field).

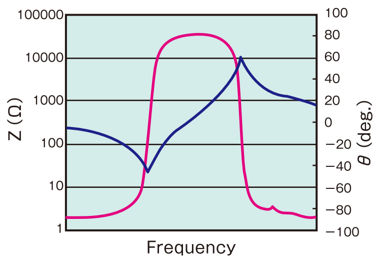
$Y_{55}{}^E$, $s_{55}{}^E$: Shear elasticity of the polarization axis (or orthogonal) direction of the plate which has electrode face in a polarization axis (at constant electric field).

■Equivalent circuit

The piezoelectric element is expressed by the equivalent circuit of FIG near the resonance frequency.



Characteristics of Impedance Z & Phase θ



C_d : Damping capacity
L_1 : Equivalent inductance
C_1 : Equivalent capacity
R_1 : Resonant resistance

The impedance and phase characteristics near the resonant frequency are shown in the figure above.

■Coupling factors k_p, k_t, k_{31}, k_{33}, k_{15}

Electromechanical coupling factor k is a factor represents the electrical and mechanical conversion capacity. It is defined as the square root of the ratio of "Arisen mechanical energy" and "Given electrical energy", or "Given mechanical energy" and "Arisen electrical energy". It is one of an amount representing the piezoelectric effect. The following is a practical formula to calculate the coupling factor k from the resonant frequency and the anti-resonance frequency.

$$1/k^2 = (a \cdot fr/\Delta f)+b,\ \Delta f = fa - fr$$

f_r : Resonance frequency

f_a : Anti-resonance frequency

Refer to the table on the right, for the coefficient a and b in the formula.

Vibration mode		a	b
Width vibration of bar plate	(k_{31})	0.405	0.595
Radial vibration of cylinder	(k_{31})	0.500	0.750
Radial vibration of disc	(k_p)	0.395	0.574
Longitudinal vibration of the rod	(k_{33})	0.405	0.810
Thickness vibration of the plate	(k_t)	0.405	0.810
Thickness shear vibration of the plate: Transversal effect	(k_{15})	0.405	0.595
Thickness shear vibration of the plate: Length effect	(k_{15})	0.405	0.810

Poisson ratio $\sigma = 0.3$

■Frequency constants N_p, N_t, N_{31}, N_{33}, N_{15}

A frequency constant N serves as length ℓ and the product with resonance frequency f_r of the corresponding direction. It's used to determine the size and the resonance frequency. The frequency constant according to vibration mode is shown by the following formula.

Radial vibration of the disc	N_p	$= f_r \cdot D$
Longitudinal vibration of the plate	N_{31}	$= f_r \cdot \ell$
Longitudinal vibration of the rod	N_{33}	$= f_r \cdot \ell$
Thickness vibration of the plate	N_t	$= f_r \cdot t$
Thickness shear vibration of the plate	N_{15}	$= f_r \cdot t$

■Dielectric constant $\varepsilon_{11}{}^T$,$\varepsilon_{33}{}^T$ & Capacitance C^T

The dielectric constant ε^T is defined by the electric displacement arising when given an electric field. Used for the analysis of the piezoelectric constant by measuring the capacitance C^T at a sufficiently lower frequency than the resonant frequency. The ratio of the permittivity ε_0 in vacuum is the relative permittivity $\varepsilon^T/\varepsilon_0$. These relationships are expressed by the following equation.

$$C^T = \varepsilon^T \cdot A/t$$

The material properties table becomes the following formula to describe the relative permittivity.

$$C^T = (\varepsilon^T/\varepsilon_0) \cdot \varepsilon_0 \cdot A/t$$

A : Electrode area [m^2] t : Interelectrode distance [m]

$\varepsilon_0 = 8.854\times10^{-12}$ [F/m]

■Piezoelectric constants d_{31}, d_{33}, d_{15}, g_{31}, g_{33}, g_{15}

Piezoelectric constant is a constant that represents the largeness of the piezoelectric effect along with the coupling factor. In the piezoelectric constants there are four constants of the piezoelectric strain constant d, piezoelectric voltage constant g and the piezoelectric stress constant e and h. Usually, d constant and g constant are used. These are defined as follows.

$$d = \frac{\text{Arisen strain}}{\text{Given field strength}} \left(\frac{\frac{m}{m}}{\frac{V}{m}} = \frac{m}{V} \right) = \frac{\text{Arisen charge density}}{\text{Given stress}} \left(\frac{\frac{C}{m^2}}{\frac{N}{m^2}} = \frac{C}{N} \right)$$

$$g = \frac{\text{Arisen field strength}}{\text{Given stress}} \left(\frac{\frac{V}{m}}{\frac{N}{m^2}} = \frac{V \cdot m}{N} \right) = \frac{\text{Arisen strain}}{\text{Given charge density}} \left(\frac{\frac{m}{m}}{\frac{C}{m^2}} = \frac{m^2}{C} \right)$$

In the definition of the previous formula, the amount of displacement with respect to the applied voltage from the d constant can be calculated. Also, from the g constant, can be calculated output voltage with respect to the applied force. e constant and h constant is in a reciprocal relation with the g constant and d constant.

■Elastic constants $Y_{11}{}^E$, $s_{11}{}^E$, $Y_{33}{}^E$, $s_{33}{}^E$, $Y_{55}{}^E$, $s_{55}{}^E$

A ratio with the longitudinal strain of the same direction as perpendicular stress is Young's modulus Y. Without considering the other direction, if a particular direction the target, handled also as elastic stiffness c. The elastic compliance s is in a reciprocal relation with the Young's modulus Y.

$$Y = c = 1/s$$

In the piezoelectric ceramics it is directly related to the frequency constant that determines the resonance frequency. And is also the amount related to the generated force.

■Poisson's ratio σ

A poisson's ratio is defined by the ratio of the transversal strain and a longitudinal strain, which arisen in perpendicular stress.

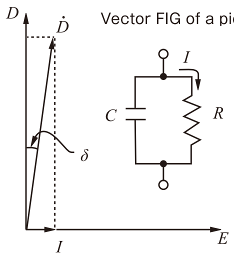
$$\sigma = \alpha/\beta = -\ s_{12}{}^E/s_{11}{}^E$$

Poisson's ratio is particularly the amount related to the resonant frequency of the binding region.

■Dielectric loss tan $\tan\delta$

When in the piezoelectric body of lossless applying a sine wave electric field E of angular frequency ω, electric displacement D vibrates in $2/\pi$ phase advanced for the electric field E. In practice, electric displacement D is δ only delayed, and arisen the loss of the phase difference. This loss will be action such as to be transformed to the dielectric heat generation. (Please refer to the following FIG.) There is a relation, as shown in the following formula to between Cd and R_1 and $\tan\delta$ of the equivalent circuit of FIG.

Vector FIG of a piezoelectric body having a loss



$$\tan\delta = 1/\omega CR$$

■Mechanical quality factor (Mechanical Q) Q_m

The piezoelectric body has elastic loss like the dielectric loss. Against stress by electric field, arising a phase difference of δ_m to the strain.

$$\tan\delta_m = 1/Q_m$$

This Q_m is the mechanical Q_m. The relation between the equivalent circuit is expressed by the following formula.

$$Q_m = 1/\omega_s C_1 R_1 = \omega_s L_1/R_1 \fallingdotseq 1/4\pi Z_r C(f_a - f_r)$$

ω_s : Angular frequency	Z_r : Resonance impedance
C_1 : Equivalent capacity	C : Capacitance
R_1 : Resonant resistance	f_r : Resonance frequency
L_1 : Equivalent inductance	f_a : Antiresonance frequency

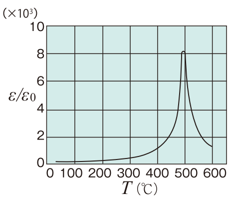
The magnitude of the Q_m, it acting on the sharpness of the mechanical vibration in the resonance frequency.

■Curie point Tc

The dielectric constant ε of the piezoelectric body, will increase to infinity with increasing temperature T. As a result, the crystal becomes unstable and will change rapidly at the temperature θ_0 at which there is a crystal system. This temperature θ_0 is the Curie point. It is the critical temperature at which completely depolarized. Piezoelectricity will be lost in this temperature. Changes in the dielectric constant ε in a high temperature range will be the relationship, such as the following formula.

$$\varepsilon = c/T - \theta_0$$

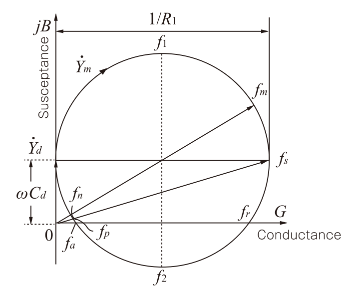
(c and θ_0 are constants)



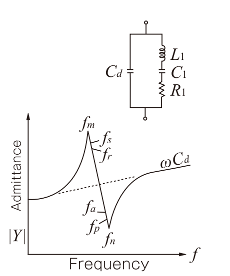
■Density ρ

The mass of the piezoelectric ceramic will be determined.

■Admittance characteristic example of the piezoceramics vibrator



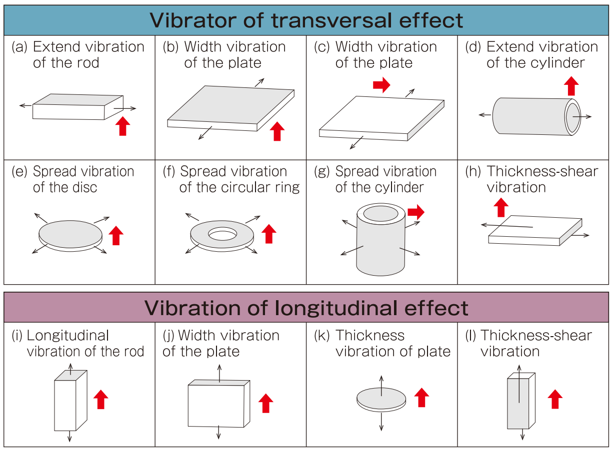
Admittance circle of piezoceramics vibrator



Admittance chart of the vibrator

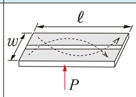
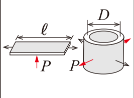
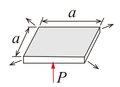
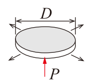
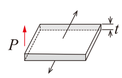
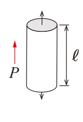
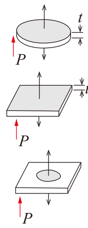
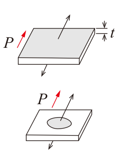
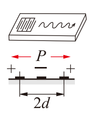
- f_s : Mechanical series resonance frequency (conductance maximum frequency)
- f_p : Mechanical parallel resonance frequency
- f_r : Resonance frequency } (susceptance=0 or a phase=0 frequency)
- f_a : Antiresonace frequency } (susceptance=0 or a phase=0 frequency)
- f_m : Frequency which the absolute value of admittance becomes the maximum
- f_n : Frequency which the absolute value of admittance becomes the minimum
- f_1 : Frequency which a susceptance becomes the maximum } quadrant Frequency
- f_2 : Frequency which a susceptance becomes the minimum } quadrant Frequency

■Examples of the vibrator shape and the vibration mode



🠉:Direction of polarization

■Basic resonance frequency of each vibrator

	Vibration mode	Vibrator shapes	Resonance frequency of fundamental vibration
Transversal effect	Bending vibration		$f_r=\dfrac{\pi}{4\sqrt{3}}\cdot\dfrac{w}{\ell^2}\cdot\sqrt{\dfrac{2}{(1+\sigma^E)}}\cdot\sqrt{\dfrac{1}{\rho s_{11}^E}}$
	Rectangular Vibration (Respiratory vibration)		$f_r=\dfrac{1}{2\ell}\sqrt{\dfrac{1}{\rho s_{11}^E}}$ D : Average diameter $f_r=\dfrac{1}{\pi D}\sqrt{\dfrac{1}{\rho s_{11}^E(1+\sigma^E)(1-\sigma^E)}}$
	Area vibration		$f_r=\dfrac{1}{2a}\sqrt{\dfrac{1}{\rho s_{11}^E(1-\sigma^E)}}$
	Radial vibration		$f_r=\dfrac{\eta_1}{\pi D}\sqrt{\dfrac{1}{\rho s_{11}^E(1+\sigma^E)(1-\sigma^E)}}$ $\eta_1=2.08$ in the case of $\sigma^E=0.35$
	Thickness shear vibration		$f_r=\dfrac{1}{2t}\sqrt{\dfrac{c_{55}^E}{\rho}}$
Longitudinal effect	Longitudinal vibration		$f_r=\dfrac{1}{2\ell}\sqrt{\dfrac{c_{33}^E}{\rho}}=\dfrac{1}{2\ell}\sqrt{\dfrac{(1-k_{33}^2)c_{33}^D}{\rho}}$ $f_a=\dfrac{1}{2\ell}\sqrt{\dfrac{1}{\rho s_{33}^D}}=\dfrac{1}{2\ell}\sqrt{\dfrac{1}{\rho s_{33}^E(1-k_{33}^2)}}$
	Thickness vibration		$f_r=\dfrac{1}{2t}\sqrt{\dfrac{c_{33}^E}{\rho}\cdot\dfrac{(1-\sigma^E)}{(1+\sigma^E)(1-2\sigma^E)}}$ $f_a=\dfrac{1}{2t}\sqrt{\dfrac{c_{33}^D}{\rho}}=\dfrac{1}{2t}\sqrt{\dfrac{c_{33}^E}{\rho(1-k_t^2)}}$
	Thickness shear vibration		$f_r=\dfrac{1}{2t}\sqrt{\dfrac{c_{55}^E}{2(1+\sigma^E)\rho}}$ $f_a=\dfrac{1}{2t}\sqrt{\dfrac{c_{55}^D}{\rho}}=\dfrac{1}{2t}\sqrt{\dfrac{1}{\rho s_{55}^E(1-k_{15}^2)}}$
	Surface acoustic wave vibration		$f_r=\dfrac{v_s}{\lambda}=\dfrac{v_s}{2d}$ v_s : Surface acoustic wave velocity

$s_{11}^D=s_{11}^E(1-k_{31}^2)$ $s_{33}^D=s_{33}^E(1-k_{33}^2)$ $c_{55}^E=c_{55}^D(1-k_{15}^2)$ $c_{55}^E=\dfrac{1}{s_{55}^E}$